

Maths skills

1 Core mathematical skills

A practical chemist must be proficient in standard form, significant figures, decimal places, SI units, and unit conversion.

1.1 Standard form

In science, very large and very small numbers are usually written in standard form. Standard form is writing a number in the format $A \times 10^x$ where A is a number from 1 to 10 and x is the number of places you move the decimal place.

For example, to express a large number such as $50\,000 \text{ mol dm}^{-3}$ in standard form, $A = 5$ and $x = 4$ as there are four numbers after the initial 5.

Therefore, it would be written as $5 \times 10^4 \text{ mol dm}^{-3}$.

To give a small number such as $0.000\,02 \text{ Nm}^2$ in standard form, $A = 2$ and there are five numbers before it so $x = -5$.

So it is written as $2 \times 10^{-5} \text{ Nm}^2$.

Practice questions

- Change the following values to standard form.
 - boiling point of sodium chloride: 1413°C
 - largest nanoparticles: $0.0\,001 \times 10^{-3} \text{ m}$
 - number of atoms in 1 mol of water: 1806×10^{21}
- Change the following values to ordinary numbers.
 - 5.5×10^{-6}
 - 2.9×10^2
 - 1.115×10^4
 - 1.412×10^{-3}
 - 7.2×10^1

1.2 Significant figures and decimal places

In chemistry, you are often asked to express numbers to either three or four significant figures. The word significant means to 'have meaning'. A number that is expressed in significant figures will only have digits that are important to the number's precision.

It is important to record your data and your answers to calculations to a reasonable number of significant figures. Too many and your answer is claiming an accuracy that it does not have, too few and you are not showing the precision and care required in scientific analysis.

For example, 6.9301 becomes 6.93 if written to three significant figures.

Likewise, 0.000 434 56 is 0.000 435 to three significant figures.

Notice that the zeros before the figure are *not* significant – they just show you how large the number is by the position of the decimal point. Here, a 5 follows the last significant digit, so just as with decimals, it must be rounded up.

Any zeros between the other significant figures are significant. For example, 0.003 018 is 0.003 02 to three significant figures.

Sometimes numbers are expressed to a number of decimal places. The decimal point is a place holder and the number of digits afterwards is the number of decimal places.

For example, the mathematical number pi is 3 to zero decimal places, 3.1 to one decimal place, 3.14 to two decimal places, and 3.142 to three decimal places.

Practice questions

- 3 Give the following values in the stated number of significant figures (s.f.).
a 36.937 (3 s.f.) **b** 258 (2 s.f.) **c** 0.043 19 (2 s.f.) **d** 7 999 032 (1 s.f.)
- 4 Use the equation:
 number of molecules = number of moles $\times$ 6.02×10^{23} molecules per mole
 to calculate the number of molecules in 0.5 moles of oxygen. Write your answer in standard form to 3 s.f.
- 5 Give the following values in the stated number of decimal places (d.p.).
a 4.763 (1 d.p.) **b** 0.543 (2 d.p.) **c** 1.005 (2 d.p.) **d** 1.9996 (3 d.p.)

1.3 Converting units

Units are defined so that, for example, every scientist who measures a mass in kilograms uses the same size for the kilogram and gets the same value for the mass. Scientific measurement depends on standard units – most are *Système International* (SI) units.

If you convert between units and round numbers properly it allows quoted measurements to be understood within the scale of the observations.

Multiplication factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n

Unit conversions are common. For instance, you could be converting an enthalpy change of $488\,889\text{ J mol}^{-1}$ into kJ mol^{-1} . A kilo is 10^3 so you need to divide by this number or move the decimal point three places to the left.

$$488\,889 \div 10^3\text{ kJ mol}^{-1} = 488.889\text{ kJ mol}^{-1}$$

Converting from mJ mol^{-1} to kJ mol^{-1} , you need to go from 10^3 to 10^{-3} , or move the decimal point six places to the left.

$$333\text{ mJ mol}^{-1}\text{ is }0.000\,333\text{ kJ mol}^{-1}$$

If you want to convert from 333 mJ mol^{-1} to nJ mol^{-1} , you would have to go from 10^{-3} to 10^{-9} , or move the decimal point six places to the right.

$$333\text{ mJ mol}^{-1}\text{ is }333\,000\,000\text{ nJ mol}^{-1}$$

Practice question

- 6 Calculate the following unit conversions.
- a** $300\text{ }\mu\text{m}$ to m
b 5 MJ to mJ
c 10 GW to kW

2 Balancing chemical equations

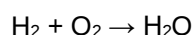
2.1 Conservation of mass

When new substances are made during chemical reactions, atoms are not created or destroyed – they just become rearranged in new ways. So, there is always the same number of each type of atom before and after the reaction, and the total mass before the reaction is the same as the total mass after the reaction. This is known as the conservation of mass.

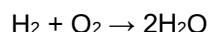
You need to be able to use the principle of conservation of mass to write formulae, and balanced chemical equations and half equations.

2.2 Balancing an equation

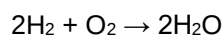
The equation below shows the correct formulae but it is not balanced.



While there are two hydrogen atoms on both sides of the equation, there is only one oxygen atom on the right-hand side of the equation against two oxygen atoms on the left-hand side. Therefore, a two must be placed before the H_2O .



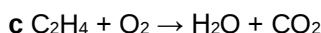
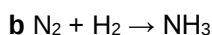
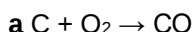
Now the oxygen atoms are balanced but the hydrogen atoms are no longer balanced. A two must be placed in front of the H_2 .



The number of hydrogen and oxygen atoms is the same on both sides, so the equation is balanced.

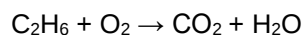
Practice question

1 Balance the following equations.

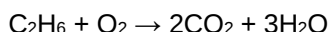


2.3 Balancing an equation with fractions

To balance the equation below:

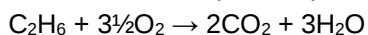


- Place a two before the CO_2 to balance the carbon atoms.
- Place a three in front of the H_2O to balance the hydrogen atoms.

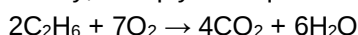


There are now four oxygen atoms in the carbon dioxide molecules plus three oxygen atoms in the water molecules, giving a total of seven oxygen atoms on the product side.

- To balance the equation, place three and a half in front of the O_2 .

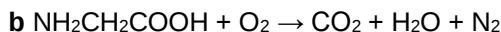
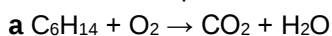


- Finally, multiply the equation by 2 to get whole numbers.

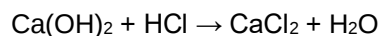


Practice question

2 Balance the equations below.

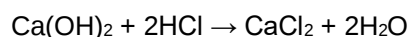


2.4 Balancing an equation with brackets



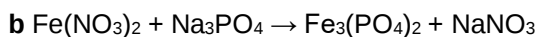
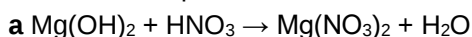
Here the brackets around the hydroxide (OH^-) group show that the $\text{Ca}(\text{OH})_2$ unit contains one calcium atom, two oxygen atoms, and two hydrogen atoms.

To balance the equation, place a two before the HCl and another before the H_2O .



Practice question

3 Balance the equations below.



3 Rearranging equations and calculating concentrations

3.1 Rearranging equations

In chemistry, you sometimes need to rearrange an equation to find the desired values.

For example, you may know the amount of a substance (n) and the mass of it you have (m), and need to find its molar mass (M).

The amount of substance (n) is equal to the mass you have (m) divided by the molar mass (M):

$$n = \frac{m}{M}$$

You need to rearrange the equation to make the molar mass (M) the subject.

Multiply both sides by the molar mass (M):

$$M \times n = m$$

Then divide both sides by the amount of substance (n):

$$M = \frac{m}{n}$$

Practice questions

1 Rearrange the equation $c = \frac{n}{V}$ to make:

a n the subject of the equation

b V the subject of the equation.

2 Rearrange the equation $PV = nRT$ to make:

a n the subject of the equation

b T the subject of the equation.

3.2 Calculating concentration

The concentration of a solution (a solute dissolved in a solvent) is a way of saying how much solute, in moles, is dissolved in 1 dm³ or 1 litre of solution.

Concentration is usually measured using units of mol dm⁻³. (It can also be measured in g dm³.)

The concentration of the amount of substance dissolved in a given volume of a solution is given by the equation:

$$c = \frac{n}{V}$$

where n is the amount of substance in moles, c is the concentration, and V is the volume in dm³.

The equation can be rearranged to calculate:

- the amount of substance n , in moles, from a known volume and concentration of solution
- the volume V of a solution from a known amount of substance, in moles, and the concentration of the solution.

Practice questions

- 3 Calculate the concentration, in mol dm⁻³, of a solution formed when 0.2 moles of a solute is dissolved in 50 cm³ of solution.
- 4 Calculate the concentration, in mol dm⁻³, of a solution formed when 0.05 moles of a solute is dissolved in 2.0 dm³ of solution.
- 5 Calculate the number of moles of NaOH in an aqueous solution of 36 cm³ of 0.1 mol dm⁻³.

4 Molar calculations

4.1 Calculating masses and gas volumes

The balanced equation for a reaction shows how many moles of each reactant and product are involved in a chemical reaction.

If the amount, in moles, of one of the reactants or products is known, the number of moles of any other reactants or products can be calculated.

The number of moles (n), the mass of the substance (m), and the molar mass (M) are linked by:

$$n = \frac{m}{M}$$

Note: The molar mass of a substance is the mass per mole of the substance. For CaCO₃, for example, the atomic mass of calcium is 40.1, carbon is 12, and oxygen is 16. So the molar mass of CaCO₃ is:

$40.1 + 12 + (16 \times 3) = 100.1$. The units are g mol⁻¹.

Look at this worked example. A student heated 2.50 g of calcium carbonate, which decomposed as shown in the equation:



The molar mass of calcium carbonate is 100.1 g mol^{-1} .

- a** Calculate the amount, in moles, of calcium carbonate that decomposes.

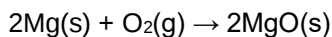
$$n = \frac{m}{M} = 2.50/100.1 = 0.025 \text{ mol}$$

- b** Calculate the amount, in moles, of carbon dioxide that forms.

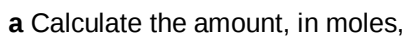
From the balanced equation, the number of moles of calcium carbonate = number of moles of carbon dioxide = 0.025 mol

Practice questions

- 1** In a reaction, 0.486 g of magnesium was added to oxygen to produce magnesium oxide.



- a** Calculate the amount, in moles, of magnesium that reacted.
b Calculate the amount, in moles, of magnesium oxide made.
c Calculate the mass, in grams, of magnesium oxide made.
- 2** Oscar heated 4.25 g of sodium nitrate. The equation for the decomposition of sodium nitrate is:



- a** Calculate the amount, in moles, of sodium nitrate that reacted.
b Calculate the amount, in moles, of oxygen made.
- 3** 0.500 kg of magnesium carbonate decomposes on heating to form magnesium oxide and carbon dioxide. Give your answers to 3 significant figures.



- a** Calculate the amount, in moles, of magnesium carbonate used.
b Calculate the amount, in moles, of carbon dioxide produced.

5 Percentage yields and percentage errors

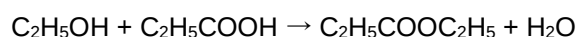
5.1 Calculating percentage yield

Chemists often find that an experiment makes a smaller amount of product than expected. They can predict the amount of product made in a reaction by calculating the percentage yield.

The percentage yield links the actual amount of product made, in moles, and the theoretical yield, in moles:

$$\text{percentage yield} = \frac{\text{actual amount (in moles) of product}}{\text{theoretical amount (in moles) of product}} \times 100$$

Look at this worked example. A student added ethanol to propanoic acid to make the ester, ethyl propanoate, and water.



The experiment has a theoretical yield of 5.00 g.

The actual yield is 4.50 g.

The molar mass of $\text{C}_2\text{H}_5\text{COOC}_2\text{H}_5 = 102.0 \text{ g mol}^{-1}$

Calculate the percentage yield of the reaction.

Actual amount of ethyl propanoate: $n = \frac{m}{M} = 4.5/102 = 0.0441 \text{ mol}$

Theoretical amount of ethyl propanoate: $n = \frac{m}{M} = 5.0/102 = 0.0490 \text{ mol}$

percentage yield = $(0.0441/0.0490) \times 100\% = 90\%$

Practice questions

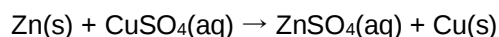
- 1 Calculate the percentage yield of a reaction with a theoretical yield of 4.75 moles of product and an actual yield of 3.19 moles of product. Give your answer to 3 significant figures.
- 2 Calculate the percentage yield of a reaction with a theoretical yield of 12.00 moles of product and an actual yield of 6.25 moles of product. Give your answer to 3 significant figures.

5.3 Calculating percentage error in apparatus

The percentage error of a measurement is calculated from the maximum error for the piece of apparatus being used and the value measured:

$$\text{percentage error} = \frac{\text{maximum error}}{\text{measured value}} \times 100\%$$

Look at this worked example. In an experiment to measure temperature changes, an excess of zinc powder was added to 50 cm^3 of copper(II) sulfate solution to produce zinc sulfate and copper.



The measuring cylinder used to measure the copper(II) sulfate solution has a maximum error of $\pm 2 \text{ cm}^3$.

- a** Calculate the percentage error.

$$\text{percentage error} = (2/50) \times 100\% = 4\%$$

- b** A thermometer has a maximum error of $\pm 0.05 \text{ }^\circ\text{C}$.

Calculate the percentage error when the thermometer is used to record a temperature rise of $3.9 \text{ }^\circ\text{C}$. Give your answer to 3 significant figures.

$$\text{percentage error} = (2 \times 0.05)/3.9 \times 100\% = 2.56\%$$

(Notice that two measurements of temperature are required to calculate the temperature change so the maximum error is doubled.)

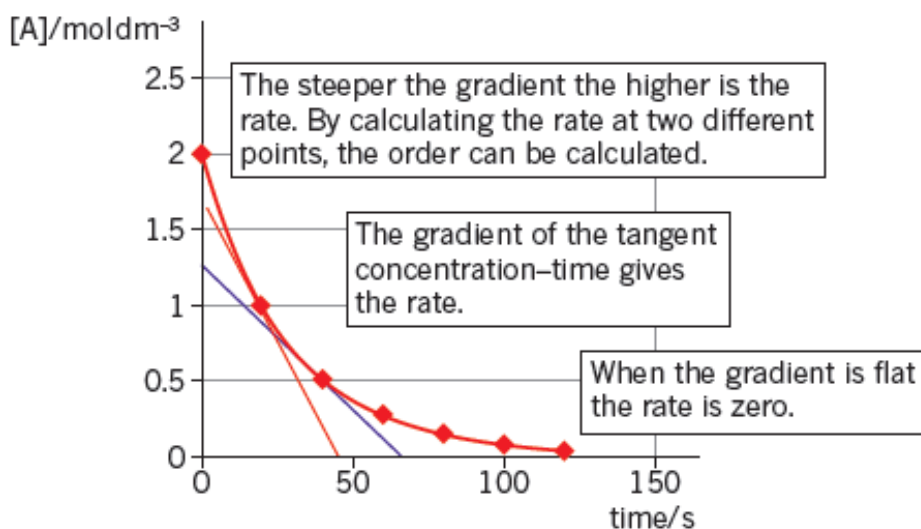
Practice questions

- 3 A gas syringe has a maximum error of $\pm 0.5 \text{ cm}^3$. Calculate the maximum percentage error when recording these values. Give your answers to 3 significant figures.
a 21.0 cm^3 **b** 43.0 cm^3
- 4 A thermometer has a maximum error of $\pm 0.5 \text{ }^\circ\text{C}$. Calculate the maximum percentage error when recording these temperature rises. Give your answers to 3 significant figures.
a $12.0 \text{ }^\circ\text{C}$ **b** $37.6 \text{ }^\circ\text{C}$

6 Graphs and tangents

6.1 Deducing reaction rates

To investigate the reaction rate during a reaction, you can measure the volume of the product formed, such as a gas, or the colour change to work out the concentration of a reactant during the experiment. By measuring this concentration at repeated intervals, you can plot a concentration–time graph.



Note: When a chemical is listed in square brackets, it just means ‘the concentration of’ that chemical. For example, $[O_2]$ is just shorthand for the concentration of oxygen molecules.

By measuring the gradient (slope) of the graph, you can calculate the rate of the reaction. In the graph above, you can see that the gradient changes as the graph is a curve. If you want to know the rate of reaction when the graph is curved, you need to determine the gradient of the curve. So, you need to plot a tangent.

The tangent is the straight line that just touches the curve. The gradient of the tangent is the gradient of the curve at the point where it touches the curve.

Looking at the graph above. When the concentration of A has halved to 1.0 mol dm^{-3} , the tangent intercepts the y-axis at 1.75 and the x-axis at 48.

The gradient is $\frac{-1.75}{48} = -0.0365$ (3 s.f.).

So the rate is $0.0365 \text{ mol dm}^{-3} \text{ s}^{-1}$.

Practice question

- Using the graph above, calculate the rate of reaction when the concentration of A halves again to 0.5 mol dm^{-3} .

6.2 Deducing the half-life of a reactant

In chemistry, half-life can also be used to describe the decrease in concentration of a reactant in a reaction. In other words, the half-life of a reactant is the time taken for the concentration of the reactant to fall by half.

Practice question

- 2 The table below shows the change in concentration of bromine during the course of a reaction.

Time / s	[Br ₂] / mol dm ⁻³
0	0.0100
60	0.0090
120	0.0066
180	0.0053
240	0.0044
360	0.0028

- a** Plot a concentration–time graph for the data in the table.
b Calculate the rate of decrease of Br₂ concentration by drawing tangents.
c Find the half-life at two points and deduce the order of the reaction.